Q1a

(a) Find the general solution to the differential equation

$$\frac{1}{2}\sec^2 3t + 2\frac{dx}{dt} = 0$$

[2]

(b) Find the particular solution to the differential equation

$$2xe^{4x} - 3\frac{dV}{dx} = 1$$

where the graph of V against x passes through the point (0,2).

$$x = \int \frac{dx}{dt}\, dt$$

a) $\frac{1}{2}\sec^2 3t + 2\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -\frac{1}{4}\sec^2 3t$$

$$x = \int -\frac{1}{4}\sec^2 3t\, dt$$

compensate, adjust

$$x = -\frac{1}{12}\int 3\sec^2 3t\, dt$$

Integrate using reverse chain rule

$$\boxed{x = -\frac{1}{12}\tan 3t + c}$$

$\int \sec^2\theta\, d\theta = \tan\theta + c$ (standard result)

Q1b

(a) Find the general solution to the differential equation

$$\frac{1}{2}\sec^2 3t + 2\frac{dx}{dt} = 0$$

[2]

(b) Find the particular solution to the differential equation

$$2xe^{4x} - 3\frac{dV}{dx} = 1$$

where the graph of V against x passes through the point (0,2).

$$V = \int \frac{dV}{dx}\, dx$$

b) $2xe^{4x} - 3\frac{dV}{dx} = 1 \Rightarrow \frac{dV}{dx} = \frac{2}{3}xe^{4x} - \frac{1}{3}$

$$V = \int\left(\frac{2}{3}xe^{4x} - \frac{1}{3}\right)dx = \int -\frac{1}{3}\,dx + \int \frac{2}{3}xe^{4x}\,dx$$

$$V = -\frac{1}{3}x + \underbrace{\frac{2}{3}\int xe^{4x}\,dx \quad u = x \quad v = \frac{1}{4}e^{4x} \quad u' = 1 \quad v' = e^{4x}}_{\text{Integration by parts}}$$

$$V = -\frac{1}{3}x + \frac{2}{3}\left(\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\right)$$

$$V = -\frac{1}{3}x + \frac{2}{3}\left(\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\right) + c$$

$$\left.\begin{aligned} V &= -\frac{1}{3}x + \frac{1}{6}xe^{4x} - \frac{1}{24}e^{4x} + c \\ V &= \frac{(4x-1)e^{4x} - 8x}{24} + c \end{aligned}\right\}\text{ general solution}$$

But when $x = 0$, $V = 2$, so

$$2 = \frac{-e^0}{24} + c = -\frac{1}{24} + c \Rightarrow c = \frac{49}{24}$$

$$\left.\boxed{V = \frac{(4x-1)e^{4x} - 8x + 49}{24}}\right\}\text{ particular solution}$$

Q2a

(a) Show that the general solution to the differential equation

$$2x\frac{\mathrm{d}y}{\mathrm{d}x} = 3kx^3y, \qquad y \neq 0$$

is

$$y = Ae^{\frac{1}{2}kx^3}$$

where A and k are constants.

[5]

(b) On separate diagrams sketch a graph of the solution for $x \geq 0$ in the instances when
(i) the constant k is greater than 0,
(ii) the constant k is less than 0.
On both diagrams state where the graph intercepts the y-axis.
You may assume $A > 0$ in both cases.

[3]

Separation of Variables

$$g(y)\frac{dy}{dx} = f(x) \Longrightarrow \int g(y)\,dy = \int f(x)\,dx$$

a)

$$2x\frac{dy}{dx} = 3kx^3y \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{3}{2}kx^2$$

$$\int \frac{1}{y}\,dy = \int \frac{3}{2}kx^2\,dx$$

$$\ln|y| = \frac{1}{2}kx^3 + c$$

$$|y| = e^{\frac{1}{2}kx^3 + c} = (e^c)(e^{\frac{1}{2}kx^3})$$

If $y > 0$, $|y| = y$, and then

$$y = e^c(e^{\frac{1}{2}kx^3})$$

$$y = Ae^{\frac{1}{2}kx^3} \quad (\text{where } A = e^c > 0)$$

If $y < 0$, $|y| = -y$, and then

$$-y = e^c(e^{\frac{1}{2}kx^3})$$

$$y = -e^c(e^{\frac{1}{2}kx^3})$$

$$y = Ae^{\frac{1}{2}kx^3} \quad (\text{where } A = -e^c < 0)$$

Q2b

(a) Show that the general solution to the differential equation

$$2x\frac{\mathrm{d}y}{\mathrm{d}x} = 3kx^3y, \qquad y \neq 0$$

is

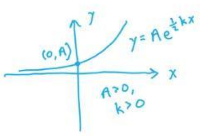
$$y = Ae^{\frac{1}{2}kx^3}$$

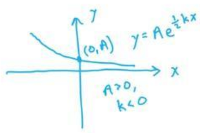
where A and k are constants.

[5]

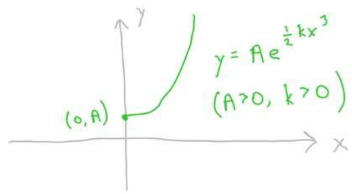
(b) On separate diagrams sketch a graph of the solution for $x \geq 0$ in the instances when
(i) the constant k is greater than 0,
(ii) the constant k is less than 0.
On both diagrams state where the graph intercepts the y-axis.
You may assume $A > 0$ in both cases.

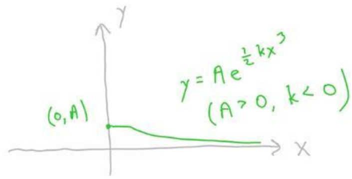
[3]





b)





Note: Compared with $y = Ae^{\frac{1}{2}kx}$, the graph of $y = Ae^{\frac{1}{2}kx^3}$ 'levels out' a bit between $x = -1$ and $x = 1$. This is because

$$-1 < x < 0 \Longrightarrow x < x^3 < 0$$

and $$0 < x < 1 \Longrightarrow 0 < x^3 < x$$

Q3

The acceleration of a particle moving in a straight line is given by the differential equation

$$\frac{d^2s}{dt^2} = 18\sin 3t - 2$$

where s m is the displacement of the particle relative to a fixed point O, and t is the elapsed time in seconds. The particle starts its journey with a velocity of -6 m s^{-1}, from a point 50 m in the positive direction from the point O.

Find an expression for the displacement, s, of the particle in terms of t.

[4]

$$\frac{ds}{dt} = \int \frac{d^2s}{dt^2}\,dt \qquad s = \int \frac{ds}{dt}\,dt$$

$$a = \frac{d^2s}{dt^2} = 18\sin 3t - 2$$

$$v = \frac{ds}{dt} = \int (18\sin 3t - 2)\,dt$$

$$v = -6\cos 3t - 2t + c_1$$

when $t = 0$, $v = -6$

$$-6 = -6\cos(0) + c_1$$

$$-6 = -6 + c_1 \implies c_1 = 0$$

$$v = -6\cos 3t - 2t$$

$$s = \int (-6\cos 3t - 2t)\,dt$$

$$s = -2\sin 3t - t^2 + c_2$$

when $t = 0$, $s = 50$

$$50 = -2\sin(0) + c_2 \implies c_2 = 50$$

$$\boxed{s = -2\sin 3t - t^2 + 50}$$

Q4

A large container of water is leaking at a rate directly proportional to the square of the volume of water in the container.

(i) Given that the initial volume of water in the container is 4000 litres and that after 10 minutes the volume of water in the container has dropped by 30%, write down and solve a differential equation connecting the volume, V, of water in the container to the time, t.

(ii) What does your solution predict will happen to the volume of water in the container after a very long time?

[8]

(i) Let V be volume in litres, and let t be time in minutes

$$\frac{dV}{dt} \propto V^2 \implies \frac{dV}{dt} = -kV^2 \quad \left(\text{where } k>0 \text{ is the constant of proportionality}\right)$$

minus because it's leaking

$$\frac{-1}{V^2}\frac{dV}{dt} = k \implies \int -\frac{1}{V^2}\,dV = \int k\,dt \implies \frac{1}{V} = kt + c$$

when $t = 0$, $V = 4000$

$$\frac{1}{4000} = k(0) + c \implies c = \frac{1}{4000}$$

when $t = 10$, $V = 0.7(4000) = 2800$

$$\frac{1}{V} = kt + c \implies k = \frac{1 - cV}{tV}$$

$$k = \frac{1 - \left(\frac{1}{4000}\right)2800}{10(2800)} = \frac{3}{280000}$$

$$\frac{1}{V} = kt + c \implies V = \frac{1}{kt + c} \text{ general solution}$$

$$V = \frac{1}{\frac{3}{280000}t + \frac{1}{4000}} \implies \boxed{V = 4000\left(\frac{70}{3t + 70}\right)}$$

particular solution (t in minutes)

Separation of Variables

$$g(y)\frac{dy}{dx} = f(x) \implies \int g(y)\,dy = \int f(x)\,dx$$

Note: If t is measured in seconds, then $k = \frac{1}{5600000}$ and the solution becomes

$$\boxed{V = 4000\left(\frac{1400}{t + 1400}\right)}$$

(ii) As t becomes large, V gets closer and closer to zero, but never actually reaches zero.

Q5a

Newton's Law of Cooling states that the rate of cooling of an object is directly proportional to the difference between the object's temperature and the ambient temperature (temperature of the object's surroundings).

(a) By setting up and solving an appropriate differential equation, show that

$$T = T_{amb} + Ae^{-kt}$$

where T °C is the temperature of the object, T_{amb} °C is the ambient temperature, t is time, and $k > 0$ and A are both constants. You may assume in working out your solution that the ambient temperature is constant, and that the temperature of the object is greater than the ambient temperature.

[6]

A meat processing factory must store its products at a temperature below −1 °C.

Due to the production process, products, before cooling, typically have a temperature between 5 °C and 10 °C.

The company therefore has a policy that any products failing to cool to below −1 °C within 6 minutes of being processed must be discarded.

The factory stores its products in a freezer with a constant ambient temperature of −4 °C.

(b) A product that has just finished being processed has a temperature of 7 °C and is immediately placed in the freezer. One minute later its temperature has dropped to 4.7 °C. Determine whether or not this product will need to be discarded.

[4]

Separation of Variables

$$g(y)\frac{dy}{dx} = f(x) \implies \int g(y)\,dy = \int f(x)\,dx$$

a)

$$\frac{dT}{dt} \propto (T - T_{amb}) \implies \frac{dT}{dt} = -k(T - T_{amb})$$

$$\frac{1}{T - T_{amb}}\frac{dT}{dt} = -k$$

$$\int \frac{1}{T - T_{amb}}\,dT = \int -k\,dt$$

$$\ln|T - T_{amb}| = -kt + c$$

$$|T - T_{amb}| = e^{-kt+c}$$

But $T > T_{amb}$, so $T - T_{amb} > 0$ and $|T - T_{amb}| = T - T_{amb}$

$$T - T_{amb} = e^{-kt+c} = (e^c)(e^{-kt})$$

$$T = T_{amb} + e^c(e^{-kt})$$

$$\boxed{T = T_{amb} + Ae^{-kt} \quad (\text{where } A = e^c)}$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$ [reverse chain rule, special case]

Q5b

Newton's Law of Cooling states that the rate of cooling of an object is directly proportional to the difference between the object's temperature and the ambient temperature (temperature of the object's surroundings).

(a) By setting up and solving an appropriate differential equation, show that

$$T = T_{amb} + Ae^{-kt}$$

where T °C is the temperature of the object, T_{amb} °C is the ambient temperature, t is time, and $k > 0$ and A are both constants. You may assume in working out your solution that the ambient temperature is constant, and that the temperature of the object is greater than the ambient temperature.

[6]

A meat processing factory must store its products at a temperature below −1 °C.

Due to the production process, products, before cooling, typically have a temperature between 5 °C and 10 °C.

The company therefore has a policy that any products failing to cool to below −1 °C within 6 minutes of being processed must be discarded.

The factory stores its products in a freezer with a constant ambient temperature of −4 °C.

(b) A product that has just finished being processed has a temperature of 7 °C and is immediately placed in the freezer. One minute later its temperature has dropped to 4.7 °C. Determine whether or not this product will need to be discarded.

[4]

b) $T_{amb} = -4 \implies T = -4 + Ae^{-kt}$

When $t = 0$, $T = 7$

$$7 = -4 + Ae^0$$

$$7 = -4 + A \implies A = 11$$

$$T = -4 + 11e^{-kt}$$

When $t = 1$, $T = 4.7$ (measuring t in minutes)

$$4.7 = -4 + 11e^{-k}$$

$$e^{-k} = \frac{8.7}{11} \implies -k = \ln\left(\frac{8.7}{11}\right)$$

$$k = -\ln\left(\frac{8.7}{11}\right) = \ln\left(\frac{11}{8.7}\right) = 0.23457\ldots$$

When $t = 6$ minutes:

$$T = -4 + 11e^{-6\ln\left(\frac{11}{8.7}\right)}$$

$$T = -1.31\ °C \quad (3\text{ s.f.})$$

Therefore the product will not need to be discarded

Alternative method for final bit of part (b)

Instead of finding the temperature of the product after 6 minutes, find out how long it takes the product to reach $-1°$ C.

$$-1 = -4 + 11e^{-\ln\left(\frac{11}{8.7}\right)t}$$

$$e^{-\ln\left(\frac{11}{8.7}\right)t} = \frac{3}{11}$$

$$-\ln\left(\frac{11}{8.7}\right)t = \ln\left(\frac{3}{11}\right)$$

$$\ln\left(\frac{8.7}{11}\right)t = \ln\left(\frac{3}{11}\right)$$

$$t = \frac{\ln(3/11)}{\ln(8.7/11)} = 5.54 \text{ minutes } (3 \text{ s.f.})$$

So the product will not need to be discarded.

Q6

Find the general solution to the differential equation

$$\frac{dy}{dx} = 2xy + 2x - y - 1, \quad y > -1$$

[5]

$$\frac{dy}{dx} = 2xy + 2x - y - 1$$

$$\frac{dy}{dx} = 2x(y+1) - (y+1) = (2x-1)(y+1)$$

$$\frac{1}{y+1}\frac{dy}{dx} = 2x - 1$$

$$\int \frac{1}{y+1}\,dy = \int (2x-1)\,dx$$

$$\ln|y+1| = x^2 - x + c$$

$$|y+1| = e^{x^2-x+c} = (e^c)(e^{x^2-x})$$

$$|y+1| = Ae^{x^2-x} \quad (A = e^c)$$

$e^c > 0$ for all c, so $A > 0$

But $y > -1$, so $y + 1 > 0$ and $|y+1| = y + 1$

$$y + 1 = Ae^{x^2-x}$$

$$\boxed{y = -1 + Ae^{x^2-x}}$$

Separation of Variables

$$g(y)\frac{dy}{dx} = f(x) \implies \int g(y)\,dy = \int f(x)\,dx$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c \quad \left[\text{reverse chain rule, special case}\right]$$

Note: If $y < -1$, then $y + 1 < 0$ and $|y+1| = -(y+1)$

In that case,

$$-(y+1) = Ae^{x^2-x}$$

$$y + 1 = -Ae^{x^2-x}$$

$$y = -1 - Ae^{x^2-x}$$

Q7a

(a) Using the standard integral result

$$\int \sec k\theta \ \mathrm{d}\theta = \frac{1}{k}\ln\left|\tan\left(\frac{k\theta}{2}+\frac{\pi}{4}\right)\right| + c$$

(where k is a constant, and c is a constant of integration), show that the solution to the differential equation

$$\cos x \frac{\mathrm{d}y}{\mathrm{d}x} = \cos y$$

with boundary condition $x = 0,\ y = \pi$, may be written in the form

$$\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

[4]

(b) (i) Show that the relationship between x and y in $\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$ may also be expressed by the set of all equations of the forms

$$y = x + 2n\pi \quad \text{or} \quad y = -x + (2n-1)\pi$$

where n is an integer.

(ii) Hence deduce that the particular solution to the differential equation in part (a), with the given boundary condition, is $y = \pi - x$.

[6]

Separation of Variables

$$g(y)\frac{dy}{dx} = f(x) \implies \int g(y)\,dy = \int f(x)\,dx$$

a)

$$\cos x \frac{dy}{dx} = \cos y \implies \frac{1}{\cos y}\frac{dy}{dx} = \frac{1}{\cos x}$$

$$\int \frac{1}{\cos y}\,dy = \int \frac{1}{\cos x}\,dx$$

$$\int \sec y\,dy = \int \sec x\,dx$$

$$\ln\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + c$$

But when $x = 0,\ y = \pi$

$$\ln\left|\tan\left(\frac{3\pi}{4}\right)\right| = \ln\left|\tan\left(\frac{\pi}{4}\right)\right| + c$$

$$\ln|-1| = \ln|1| + c$$

$$\ln(1) = \ln(1) + c$$

$$0 = 0 + c \implies c = 0$$

So

$$\ln\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

$$\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

Take exponential of both sides to get rid of the logarithms

Q7b

[6]

Facts about tangent function

① $\tan(-\theta) = -\tan\theta$

② $\tan\theta$ is periodic and repeats every π radians (b)(i)

$|a| = |b| \Rightarrow a = \pm b$

$\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| \Rightarrow \tan\left(\frac{y}{2}+\frac{\pi}{4}\right) = \pm\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$

If $\tan\left(\frac{y}{2}+\frac{\pi}{4}\right) = \tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$

$\frac{y}{2}+\frac{\pi}{4} = \left(\frac{x}{2}+\frac{\pi}{4}\right) + n\pi$ (where n is an integer)

$\frac{y}{2} = \frac{x}{2} + n\pi$ ← Fact ②

$y = x + 2n\pi$

If $\tan\left(\frac{y}{2}+\frac{\pi}{4}\right) = -\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$

$\tan\left(\frac{y}{2}+\frac{\pi}{4}\right) = \tan\left(-\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)$ ← Fact ①

$\frac{y}{2}+\frac{\pi}{4} = -\left(\frac{x}{2}+\frac{\pi}{4}\right) + n\pi$ (where n is an integer) ← Fact ②

$\frac{y}{2}+\frac{\pi}{4} = -\frac{x}{2} - \frac{\pi}{4} + n\pi$

$\frac{y}{2} = -\frac{x}{2} - \frac{\pi}{2} + n\pi$

$y = -x - \pi + 2n\pi$

$y = -x + (2n-1)\pi$

(a) Using the standard integral result

$$\int \sec k\theta \, d\theta = \frac{1}{k}\ln\left|\tan\left(\frac{k\theta}{2}+\frac{\pi}{4}\right)\right| + c$$

(where k is a constant, and c is a constant of integration), show that the solution to the differential equation

$$\cos x \frac{dy}{dx} = \cos y$$

with boundary condition $x = 0$, $y = \pi$, may be written in the form

$$\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$$

[4]

(b) (i) Show that the relationship between x and y in $\left|\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)\right| = \left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|$ may also be expressed by the set of all equations of the forms

$$y = x + 2n\pi \quad \text{or} \quad y = -x + (2n-1)\pi$$

where n is an integer.

(ii) Hence deduce that the particular solution to the differential equation in part (a), with the given boundary condition, is $y = \pi - x$.

(b)(ii)

By the boundary condition, $y = \pi$ when $x = 0$.
This is never true for equations of the form

$y = x + 2n\pi$

where n is an integer.

It is only true for equations of the form

$y = -x + (2n-1)\pi$

if $n = 1$. In that case,

$y = -x + \pi$

$y = \pi - x$

Q8a

A tree disease is spreading throughout a large forested area.

The rate of increase in the number of infected trees is modelled by the differential equation

$$\frac{dN}{dt} = kN(N-1), \quad N > 1$$

where N is the number of infected trees, t is the time in days since the disease was first identified and k is a positive constant.

(a) Solve the differential equation above, and show that the general solution can be written in the form

$$N = \frac{1}{1 - Ae^{kt}}$$

where A is a positive constant.

[6]

(b) Initially two trees were identified as diseased.
A fortnight later, 4 trees were infected.
Using this information, find the values of the constants A and k.

[3]

(c) By considering the solution to the differential equation along with the values of A and k found in part (b), suggest a range of values of t for which the model might be considered reliable.

[3]

Separation of Variables

$$g(y)\frac{dy}{dx} = f(x) \implies \int g(y)\,dy = \int f(x)\,dx$$

a)

$$\frac{dN}{dt} = kN(N-1) \implies \frac{1}{N(N-1)}\frac{dN}{dt} = k$$

$$\int \frac{1}{N(N-1)}\,dN = \int k\,dt$$

But $\frac{1}{N(N-1)} = \frac{1}{N-1} - \frac{1}{N}$ (partial fractions)

$$\int \left(\frac{1}{N-1} - \frac{1}{N}\right) dN = \int k\,dt$$

$$\ln|N-1| - \ln|N| = kt + c$$

$$\ln\frac{|N-1|}{|N|} = kt + c$$

But $N > 1$, so $N-1 > 0$, $N > 0 \implies |N-1| = N-1$, $|N| = N$

$$\ln\frac{N-1}{N} = kt + c$$

$$\frac{N-1}{N} = e^{kt+c} = (e^c)(e^{kt}) = Ae^{kt} \quad (A = e^c > 0)$$

Because $e^c > 0$ for any value of c

$$N - 1 = N(Ae^{kt})$$

$$N - N(Ae^{kt}) = 1$$

$$N(1 - Ae^{kt}) = 1$$

$$\boxed{N = \frac{1}{1 - Ae^{kt}}}$$

Q8b

b) When $t = 0$, $N = 2$

$$2 = \frac{1}{1 - Ae^0} = \frac{1}{1-A}$$

$$2 - 2A = 1 \implies 2A = 1 \implies \boxed{A = \frac{1}{2}}$$

When $t = 14$, $N = 4$

$$4 = \frac{1}{1 - \frac{1}{2}e^{14k}}$$

$$4 - 2e^{14k} = 1$$

$$e^{14k} = \frac{3}{2}$$

$$14k = \ln\left(\frac{3}{2}\right)$$

$$\boxed{k = \frac{1}{14}\ln\left(\frac{3}{2}\right)} \approx 0.0290 \text{ (3 s.f.)}$$

Q8c

A tree disease is spreading throughout a large forested area.

The rate of increase in the number of infected trees is modelled by the differential equation

$$\frac{dN}{dt} = kN(N-1), \qquad N > 1$$

where N is the number of infected trees, t is the time in days since the disease was first identified and k is a positive constant.

(a) Solve the differential equation above, and show that the general solution can be written in the form

$$N = \frac{1}{1 - Ae^{kt}}$$

where A is a positive constant.

If $1 - Ae^{kt} = 0$, N is undefined.
If $1 - Ae^{kt} < 0$, N is negative, and you can't have a negative number of trees!

[6]

(b) Initially two trees were identified as diseased.
A fortnight later, 4 trees were infected.
Using this information, find the values of the constants A and k.

[3]

(c) By considering the solution to the differential equation along with the values of A and k found in part (b), suggest a range of values of t for which the model might be considered reliable.

[3]

From part (b), $A = \frac{1}{2}$ and $k = \frac{1}{14}\ln\left(\frac{3}{2}\right)$

c) The solution is only valid if

$$1 - Ae^{kt} > 0$$
$$Ae^{kt} < 1$$
$$e^{kt} < \frac{1}{A}$$
$$kt < \ln\left(\frac{1}{A}\right)$$
$$t < \frac{1}{k}\ln\left(\frac{1}{A}\right)$$
$$t < \frac{1}{\frac{1}{14}\ln(3/2)}\ln\left(\frac{1}{1/2}\right)$$

$$\boxed{t < \frac{14\ln(2)}{\ln(3/2)} \approx 23.9 \text{ days (3 s.f.)}}$$

The model is likely to be reliable for values of t between 0 and 23 days.